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CERN/LEP-TH/88-05

User's Guide for New MOSES Version 2.0 (MOde-coupling Single bunch instability in an Electron Storage ring)

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April 26, 1988

Abstract

MOSES is a computer program which computes (complex)coherent betatron tune shifts as a function of the bunch current for a Gaussian beam and provides their graphical representation on a line printer. The new version of MOSES presented in this note is different in two respects from the old one: (1) the betatron tune may have a spread, (2) the synchrotron tune can be varied as a function of current as the user wishes. The method of calculation in the presence of betatron tune spread can be chosen to be either the author's dispersion integral method or Besnier's dispersion matrix method. The outline of these methods is explained in the Appendix. MOSES is available as source code in both the CERN-IBM VM/CMS and the MVS/Wylbur System.

1 Introduction

A bunched beam in a storage ring interacts with its environment electromagnetically by exciting wake fields. These fields kick the beam, and the oscillation frequency of coherent particle motion may be changed. When the frequencies of two different transverse coherent motions coincide, one of those motions will become unstable, with a growth rate comparable to the synchrotron frequency. This is called a transverse mode-coupling instability[1,2]. MOSES (MOde-coupling Single bunch instability in an Electron Storage ring) is a computer program which computes oscillation tunes of transverse coherent motion as a function of the bunch current for a Gaussian beam, and provides their graphical representation on a line printer. The first version of MOSES was published in 1986[3]. Since then, it has been shown in several publications that the program gives good agreement with computer simulations[4]. MOSES was installed to the program BBI by Zotter and Gygi-Hanney[5], and can be called from the inside of BBI or can be used as a stand-alone program. The new version of MOSES presented in this note has the following two differences from the old version:

1. The betatron tune may have a spread.

2. The synchrotron frequency can be varied with the bunch current as the user wishes.

The method of calculation in the presence of betatron tune spread can be chosen to be either the author's dispersion integral method[6] or Besnier's dispersion matrix method[7]. Their outline is explained in the Appendix. Details of the formalism can be found in Refs.(6-8). In the former method, solutions are obtained by finding zeros of the determinant of a certain complex matrix. This method is valid only when the zeros have non-zero imaginary parts. Stationary solutions cannot be calculated correctly. However, some users might want to guess where the tunes of stationary modes will appear, even if it is rough guess. MOSES provides such a guess, by assuming that a stationary solution may be approximated roughly by the purely real value which gives a local minimum to the absolute value of the determinant, and by searching such a point with a minimization program. Sometimes the program fails to find local minima for large tune spreads, mainly because they are not well defined or do not exist. On the contrary, solutions in the latter method are obtained as eigenvalues of the complex matrix as in the absence of tune spread, and hence are determined without uncertainty. But this requires a rather large matrix size ($\sim 100 \times 100$) for accurate solutions, and therefore is very time consuming (actually solutions converge to those obtained with the dispersion integral method in the limit of infinite matrix size). A combined use of the two methods is therefore recommended. We discuss this problem again in Sec.4.

There are some new variables related to the above differences. There is no change in the old variables, so the user can still use input data for the old version. These old variables are already explained in Ref.3. However, for the convenience of the reader, I shall try to make this note serve as a full user's guide by explaining all input variables, even if many of them may have been described in Ref.3.

The output of MOSES contains, in addition to the printout of all input parameters, complex values of the coherent tune shifts of a specified number of modes, and their

graphical representation on a line printer. In the following sections, we will explain the definitions of the input variables and how to interpret the output, referring to an example for a test job. The final section is devoted to some remarks on how to search coherent tune shifts in the presence of betatron tune spread and how to interpret the results.

2 Input variables

A sample input data for the electron storage ring DCI of Orsay is given in Fig.1. It consists of five **NAMELIST** formats and one headline. If you are a user of the CERN IBM VM/CMS, you must have access to the LEPTH disk, where you will find a copy of the files: MOSES FORTRAN (source code), MOSES EXEC (EXEC file for executing MOSES), MOSES HELPCMS file and a test data file TEST MOSDATA.
If you don't know about the LEPTH disk ....
You need log in to CERNVM and type `GIME PMG 195` to obtain access to the disk. Then the command `LEPTHDSK` will allow you to take full advantage of the software on the disk, including permanent access if you wish. This command will also guide you to the on-line HELP information which includes help for using MOSES.
In order to run the program, you must have a data file on the A-disk, called fn MOSDATA. If fn is not specified, it is assumed to be TEST. (For test purposes you could copy the TEST MOSDATA file from the LEPTH disk). Then type in:

```
MOSES fn
```

The results will be on the file MOSES RESULTS on your A-disk.

If you are a user of the MVS/Wylbur, you can get the same input data with the JCL (job control card) by typing in Wylbur mode

```
EXEc FROm $IZ.ZOT.LIB#MOSESTST
```

and answering the questions. We list all input variables with brief explanations, their units or formats, and default values in what follows.

TITLE FORMAT(A72)

The title of the program run which may be printed as header to the program. If you do not want a title, leave it blank, but never eliminate the line.

NAMELIST `&MPARM` defines machine and beam parameters.

`NUS` Synchrotron tune.
If `NUS` $>$ `0`, the synchrotron tune is kept constant as input while the bunch current varies.

		If NUS = 0, the synchrotron tune varies with the bunch current according to the user specified arrays of the synchrotron tunes NUSTOC(i) and the bunch currents CTONUS(i) (see also the explanation of NUSTOC and CTONUS for more details.) If NUS < 0, the synchrotron tune varies with the bunch current according to a user defined function. The user must supply a synchrotron tune function into the source program. Copy the source file into your file, and edit the subroutine function FNUS(X) following the instruction written there.
ENGY	GeV	Beam energy.
SGMZ	cm	Bunch length. If SGMZ > 0, the bunch length kept constant as input while the bunch current varies. If SGMZ = 0, the bunch length varies with the bunch current according to the user specified arrays of the bunch lengths SGMTOC(i) and the bunch currents CTOSGM(i) (see also the explanation of SGMTOC and CTOSGM for more details.) If SGMZ < 0, the bunch length varies with the bunch current according to a user defined function. The user must supply a bunch length function into the source program. Copy the source file into your file, and edit the subroutine function FSGM(X) following the instruction written there.
BETAC	meter	Beta-function at the location of the impedance.
REVFRQ	MHz	Revolution frequency of a reference particle.
ALFA		Momentum compaction factor.
CHROM	(default=0)	Linear chromaticity defined by $d\nu_x/dp/p$.
SPRD	(default=0)	Betatron tune spread at one standard deviation of bunch length. A parabolic amplitude dependence of tune spread is assumed, namely $\nu_x = \nu_{x0} - \text{SPRD} \cdot (\frac{x_\beta}{\sigma_x})^2$. See also the Appendix.
NUSTOC(i), CTONUS(i)	mA ($i \leq 120$)	When NUS = 0, these variables are used: NUSTOC(i) is the array to store the synchrotron tunes at the bunch currents CTONUS(i). The synchrotron tune between two points CTONUS(i) and CTONUS(i+1) is calculated by linear interpolation.
SGMTOC(i),	cm	When SGMZ = 0, these variables are used:

CTOSGM(i)	mA ($i \leq 120$)	SGMTOC(i) is the array to store the bunch lengths at the bunch currents CTOSGM(i). The bunch length between two points CTOSGM(i) and CTOSGM(i+1) is calculated by linear interpolation.
NAMELIST	&IPARM	defines parameters of the transverse impedance. A resonator model is assumed for the impedance.
FREQ	MHz	Resonant frequency of the impedance.
RS	MΩ/m	Impedance at the resonant frequency.
QV		Quality factor.
NAMELIST	&CPARM	parameters relevant to the computation of coherent tune shifts and control card for printout.
CRNT	mA	The first bunch current with which the computation starts. The value must be larger than zero.
STPC	mA	Step on increase in bunch current. The bunch current varies as CRNT, CRNT+STPC, ..., CRNT + STPC × (NCR-1). Can be negative.
NCR	(default=60)	Number of steps on change in bunch current to be executed. Must be less than 120.
NMODE	(default=-2)	The lowest azimuthal mode number to be included in the calculation.
NMODF	(default=1)	The highest azimuthal mode number to be included in the calculation.
KRAD	(default=0)	Number of higher longitudinal radial modes to be used for expansion of the radial function for each azimuthal mode. Note that zero means that each azimuthal mode has one radial mode.
LPRINT	Logical*1 (default=T)	When LPRINT = .TRUE., numerical values of coherent tune shifts normalized by the synchrotron tune are printed with the bunch length and the synchrotron tune for each bunch current.
NAMELIST	&HPARM	defines the window of graphical output of coherent tune shifts on a line printer.

MMAX	(default= NMODF)	The maximum coordinate of the figure of the real part of coherent tune shifts.
MMIN	(default= NMODE)	The minimum coordinate of the figure of the real part of coherent tune shifts.
TAUMAX	(default=0.1)	The maximum coordinate of the figure of the imaginary part of coherent tune shifts.
TAUMIN	(default=-0.1)	The minimum coordinate of the figure of the imaginary part of coherent tune shifts.
NSPL	(default=1)	Number of split pictures of the real part of coherent tune shifts.

NAMELIST	&SPARM	defines parameters necessary for searching coherent tune shifts in the presence of betatron tune spread. If SPRD = 0, this **NAMELIST** is not read. See also remarks in Sec.4.
LMTRIX	Logical*1 (default=F)	When LMTRIX = .TRUE., Besnier's dispersion matrix method is used. The user has to specify NDISP, number of higher transverse radial modes for expansion of the transverse dipole distribution.
LINTGL	Logical*1 (default=T)	When LINTGL = .TRUE., the dispersion integral method is used. The user has to specify all variables from NSTPS down to CXMAX described in this **NAMELIST**. Solutions are identified as points which give zeros or local minima of the absolute value of the determinant of the matrix **A** defined in the Appendix. The simplex method and the Müller method are used for minimum search. For a large tune spread, it is recommended to increase the tune spread gradually from a small value to the desired one, using NSTPS and SPRDO parameters.
LMAP	Logical*1 (default=F)	When LMAP = .TRUE., the absolute value of the determinant of the matrix **A** defined in the Appendix is plotted on line printer as a function of real tune shift in a interval (MMIN,MMAX) with increment 0.05 between points. The subroutine MAP in CERN GENLIB is also used for plotting. See also remarks in Sec.4.
NDISP	(default=9)	Number of higher transverse radial modes for expansion of the dipole particle distribution. Used

		only when LMTRIX = .TRUE.
NSTPS	(default=1)	Number of steps for changing the tune spread. Must be less than 10 on VM and 6 on MVS. If the starting tune spread, SPRD0, is not zero, NSTPS has to be larger than two. The tune spread at each step is : SPRD$\times i$/NSTPS, $i = 1$ to NSTPS, when SPRD0 = 0, SPRD0 + (SPRD-SPRD0)$\times(i-1)$/(NSTPS-1), $i = 1$ to NSTPS, when SPRD0 > 0.
SPRD0	(default=0)	Starting betatron tune spread.
EX	(default=0.1)	Initial search step for the real part of the tune shifts. Search is done in both directions EX and -EX.
EY	(default=0.01)	Initial search step for the imaginary part of the tune shifts.
FEPS	(default=1.D-10)	Criterion for convergence of the function value.
ZEPS	(default=0)	Criterion for convergence of the solution. When zero, this criterion will be ignored.
DXMAX	(default=0.5)	The maximum difference of the real tune shift in which the initial estimate of solutions is taken from the results for one current step smaller. See also remarks in Sec.4.
ESPC	(default=0.001)	The minimum distance between different solutions in the complex plane. If their distance is within this value, they are considered to be the same solution.
CXMAX	(default=0.5)	The maximum change in real tune shift which can be accepted as a solution for the current one step larger. If the change is larger than this value, that solution is neglected.

3 Output of MOSES

The output for the test data is shown in Fig.2. We will briefly explain how to interpret this for the reader's convenience. The beginning of the output is signalled by the title of the program followed by the date, time and the version number of MOSES. All the input variables are printed with short explanations of their definitions and with their units. Next follows the printout of the bunch length and the synchrotron tune as a function of the bunch current if they vary with the current. When SGMZ(NUS) = 0, the content of the arrays SGMTOC and CTOSGM (NUSTOC and CTONUS) will be printed. If LPRINT = .TRUE., complex values of all coherent tune shifts, normalized by the synchrotron tune, are printed for each bunch current together with the bunch length and the synchrotron tune. If SPRD $\neq 0$ and LINTGL = .TRUE., the betatron tune spread is printed at the head of printout.

The normalized coherent tune shifts are plotted at separate pages by line printer. The window of these figures are defined in **NAMELIST** $HPARM. The title of the job, date,

time and the tune spread are printed in the header line. If SPRD $\neq 0$, the method used for finding solutions is also typed.

4 Some remarks

In this final chapter, we add some remarks on the computation algorithm and on the interpretation of results for the case of nonzero betatron tune spread. First, we consider the case where the dispersion integral method is used. There is no uniquely defined way to to calculate coherent tune shifts for stationary modes, only when the imaginary part of tune shifts becomes nonzero, solutions can be rigorously obtained. This corresponds to the fact that there is no method of the mapping from the complex tune-shift plane to the stable region in the stability diagram. In MOSES, it is assumed that such a stationary solution may be approximated by a purely real tune which corresponds a local minimum of the absolute value of the determinant of the matrix **A** (see Appendix for its definition). This assumption is reasonable, since zeros of this determinant are true eigensolutions when the tune spread is zero. Therefore, one may expect that this function will still have well-defined local minima around solutions as long as the tune spread is small. However, it is not clear how good or bad this approximation is for large tune spreads. There is even no physical foundation for the existence of local minima. The user has to keep in mind that tunes of stationary solutions provided by MOSES have meaning only as a guide for the user's convenience. When MOSES fails to find stationary solutions, this might be due to the nature of the problem.

The parameter **DXMAX** is related to the initial estimate of solutions. There are two ways to set the initial estimate: from the result for a smaller tune spread at the same current, or from the result for a smaller current at the same tune spread. MOSES decides which value should be taken through a rather complicated algorithm: First, it takes the initial value from the solution for the tune spread one step smaller (if NSTPS = 1, from the solution for no tune spread) at the same current. We shall call it X0. If there is a solution within $\pm$**DXMAX** around X0 for the current one step smaller, at the same tune spread, the program replaces X0 by this value. In this process, any two solutions whose distance is less than **ESPC** in complex tune space are supposed be the same solution, and one of them is not therefore used as an initial estimate of the solution for the next current. If the solution from the minimization program lies further away from the initial estimate than **CXMAX**, it is rejected.

Sometimes the minimization program converges to the same local minimum or fails to find a local minimum at all. If this happens, the following recipe might be worth to try:

- Set **STPC** negative, and compute backwards from the largest current.
- Try a finer step size for changing the tune spread, using **NSTPS**.
- Change the parameters **DXMAX**, **CXMAX**, **EX**,**EY**,**FEPS**, and **ZEPS**.
- If the m=0 mode is not found, set **NMODF** to a larger value.

If solutions cannot be found even after all these tries, and if the user is interested in finding all local minima if they exist, he should use the **LMAP** parameter. It gives a graphical picture of the function to be minimized, as function of the real tune shift in the interval (**MMIN**,**MMAX**) with distance 0.05 between neighboring points. The user may then find what is going on by his inspection.

Next, we consider the Besnier's dispersion matrix method. This method has the advantage that all solutions are always obtained, no matter whether solutions are stationary or unstable. It has, however, the following two drawbacks: (1) We need a large number of orthogonal polynomials to expand the transverse dipole distribution for accurate calculation. A suitable **NDISP** is around **SPRD** $\times 500$ / **NUS**. (2) There is no stability threshold. This might be explained physically as follows. In this method, a infinite set of δ functions or shell distributions, each having a different tune determined by its phase space amplitude, is approximated by a finite set of orthogonal coherent distributions. In reality, Landau damping is due to the fact that the energy of unstable coherent oscillations is absorbed by those shell distributions whose oscillation frequency resonates with the coherent one. This situation is now described by only a few orthogonal distributions. Therefore the share of each orthogonal distribution becomes relatively large, and they will always remain slightly unstable in order to suppress a strongly unstable mode. In this method, the number of eigensolutions will be quite large. It will be helpful to split the picture for the real parts of the coherent tune shift by using the **NSPL** parameter.

Acknowledgement

The author would like to thank B. Zotter for proposing the problem and for helpful discussions, and P.M.Gygi-Hanney for her support with the CERN-IBM VM/CMS System. He also would like to thank E. Keil and the LEP TH Group for its hospitality.

Appendix: The dispersion integral and Besnier's dispersion matrix methods

In this appendix, we outline the two methods used for calculation in the presence of betatron tune spread. The definition of notations follows those in Ref.1.

The equation which we want to solve has the form of an integral equation for particle distribution:

$$[\nu - m\nu_s - \nu_x(r_x)]f(r_x)g_m(r_s) = -\nu_s D\frac{df_0}{dr_x}w_s(r_s)\sum_n \int_0^\infty G_n^m(r_s, r_s')g_n(r_s')dr_s', \qquad (1)$$

where $f(r_x)$ and $g_m(r_s)$ are the perturbed transverse and longitudinal distributions as a function of the phase space amplitudes r_x and r_s, respectively, and ν is the coherent tune to be determined. We assume that the betatron tune ν_x is a function of r_x only and can be expressed as

$$\nu_x = \nu_{x0} - S_x(r_x). \qquad (2)$$

The explicit form of the kernel $G_n^m(r_s, r_s')$ can be derived from Eq.(2.22) of Ref.1. The transverse dipole moment D is given by

$$D = \pi \int_0^\infty f(r_x) r_x^2 dr_x. \tag{3}$$

The other notations are as follows: ν_s is the synchrotron tune, $f_0(r_x)$ is the transverse unperturbed distribution, and $w_s(r_s)$ is the longitudinal weight function. First we show how to solve Eq.(1) according to the dispersion integral method.

If we divide both sides of Eq.(1) by $(\nu - m\nu_s - \nu_x(r_x))$, multiply by r_x^2, and integrate over r_x, we obtain

$$g_m(r_s) = F_m(\nu) w_s(r_s) \sum_n \int_0^\infty G_n^m(r_s, r_s') g_n(r_s') dr_s', \tag{4}$$

where

$$F_m(\nu) = -\nu_s \pi \int_0^\infty \frac{\frac{df_0}{dr_x} r_x^2 dr_x}{\nu - m\nu_s - \nu_x(r_x)}. \tag{5}$$

We solve Eq.(4) by expanding the unknown function $g_m(r_s)$ into normalized orthogonal polynomials which satisfy the relation

$$\int_0^\infty w_s(r_s) h_k^{(m)}(r_s) h_l^{(n)}(r_s) r_s dr_s = \delta_{mn}\delta_{kl}. \tag{6}$$

The kernel can also be expanded with the same orthogonal polynomials. The results are:

$$g_m(r_s) = w_s(r_s) \sum_{k=0}^\infty a_k^{(m)} h_k^{(m)}(r_s), \tag{7}$$

$$G_n^m(r_s, r_s') = \sum_{k,l} M_{nl}^{mk} h_k^{(m)}(r_s) h_l^{(n)}(r_s'). \tag{8}$$

The explicit form of the matrix element M_{nl}^{mk} is given by Eq.(2.44) in Ref.1. Multiplying Eq.(4) by $h_k^{(m)}(r_s) r_s$ and integrating over r_s, we obtain

$$a_k^{(m)} = F_m(\nu) \sum_{n=-\infty}^\infty \sum_{l=0}^\infty M_{nl}^{mk} a_l^{(n)}. \tag{9}$$

The set of equations has a non-trivial solution only if

$$det\mathbf{A} = 0, \tag{10}$$

where $\mathbf{A}$ is the matrix with elements

$$A_{nl}^{mk} = \delta_{mn}\delta_{kl} F_m^{-1}(\nu) - M_{nl}^{mk}. \tag{11}$$

The condition(10) may be called a generalized dispersion relation.

If $f_0(r_x)$ is a Gaussian distribution with a beam size σ_x:

$$f_0(r_x) = \frac{1}{2\pi\sigma_x^2}\exp(-\frac{r_x^2}{2\sigma_x^2}) \tag{12}$$

and the tune spread $S_x(r_x)$ has a parabolic dependence on r_x:

$$S_x(r_x) = S(\frac{r_x}{\sigma_x})^2, \tag{13}$$

the integration of F_m is achieved readily, with the result

$$F_m(\nu) = \frac{\nu_s}{2S}[1 - \tau e^{\tau}\mathrm{E}_1(\tau)], \tag{14}$$

where $\mathrm{E}_1(x)$ is the exponential integral[9], and

$$\tau = \frac{\nu - m\nu_s - \nu_{x0}}{2S}. \tag{15}$$

Next, we solve Eq.(1) by Besnier's dispersion matrix method. In this method, we expand, not only the longitudinal distribution $g_m(r_s)$, but also the transverse distribution $f(r_x)$. Using the orthogonal polynomials $d_s(r_x)$ which satisfy

$$\int_0^\infty w_x(r_x)d_s(r_x)d_t(r_x)r_x dr_x = \delta_{st}, \tag{16}$$

where $w_x(r_x)$ is the transverse weight function defined by

$$w_x(r_x) = K\frac{1}{r_x}\frac{df_0(r_x)}{dr_x}, \tag{17}$$

with the normalizing factor K, $f(r_x)$ can be expanded as

$$f(r_x) = w_x(r_x)\sum_{s=0}^{\infty} c_s d_s(r_x). \tag{18}$$

We expand also the product of $S_x(r_x)$ and $f(r_x)$ in the same orthogonal polynomials:

$$S_x(r_x)f(r_x) = w_x(r_x)\sum_{s=0}^{\infty} b_s d_s(r_x), \tag{19}$$

where the coefficients b_s are related to the coefficients c_t by

$$b_s = \sum_{t=0}^{\infty} N_{st}c_t, \tag{20}$$

where

$$N_{st} = \int_0^\infty S_x(r_x)w_x(r_x)d_s(r_x)d_t(r_x)r_x dr_x \tag{21}$$

is called the dispersion matrix.

The transverse dipole moment D can now be expressed with the coefficients c_s by inserting Eq.(18) into Eq.(3), with the result

$$D = \pi \sum_s c_s \int_0^\infty w_x(r_x) d_s(r_x) r_x^2 dr_x. \tag{22}$$

If one uses the fact that the lowest polynomial is proportional to r_x:

$$d_0(r_x) = \kappa r_x, \tag{23}$$

one finds that only the lowest coefficient in the summation remains. Therefore,

$$D = \pi \frac{c_0}{\kappa}. \tag{24}$$

If we insert Eqs.(17-20) and (24) into Eq.(1) and notice that $-\pi\kappa^2/K$ is one, Eq.(1) can be rewritten as

$$\begin{aligned} & [(\nu - m\nu_s - \nu_{x0}) w_x(r_x) \sum_s c_s d_s(r_x) + w_x(r_x) \sum_{s,t} N_{st} c_t d_s(r_x)] g_m(r_s) \\ = \;& \nu_s w_x(r_x) c_0 d_0(r_x) \times w_s(r_s) \sum_n \int_0^\infty G_n^m(r_s, r_s') g_n(r_s') dr_s'. \end{aligned} \tag{25}$$

By inserting Eq.(8) into Eq.(25), multiplying by $h_k^{(m)}(r_s) r_s \cdot d_s(r_x) r_x$, and integrating over r_s and r_x, we obtain

$$[(\nu - m\nu_s - \nu_{x0}) c_s + \sum_{t=0}^{\infty} N_{st} c_t] a_k^{(m)} = \nu_s c_0 \delta_{s0} \sum_{n=-\infty}^{\infty} \sum_{l=0}^{\infty} M_{nl}^{mk} a_l^{(n)}. \tag{26}$$

If one defines new coefficients

$$e_{ks}^{(m)} = c_s a_k^{(m)}, \tag{27}$$

these coefficients have a non-trivial solution only if

$$det(\lambda \mathbf{I} - \mathbf{B}) = 0, \tag{28}$$

where

$$\lambda = \frac{\nu - \nu_{x0}}{\nu_s} \tag{29}$$

is the normalized tune shift, $\mathbf{I}$ is the unit matrix, and $\mathbf{B}$ is the matrix with elements

$$B_{tnl}^{smk} = m\delta_{st}\delta_{mn}\delta_{kl} - \frac{N_{st}}{\nu_s}\delta_{mn}\delta_{kl} + M_{nl}^{mk}\delta_{s0}\delta_{t0}. \tag{30}$$

For a $f_0(r_x)$ of a Gaussian distribution, and S_x of Eq.(13), elements of the dispersion matrix are written explicitly as

$$N_{kl} = 2S[2(k+1)\delta_{kl} - \sqrt{(k+1)(l+1)}\delta_{k+1,l} - \sqrt{(k+1)(l+1)}\delta_{k-1,l}]. \tag{31}$$

References

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```
D C I  OF  O R S A Y - SAMPLE INPUT DATA FOR MOSES
 &MPARM
 NUS=0.00792D0, ENGY=.8D0, SGMZ=-1.D0, BETAC=8.86D0
 REVFRQ=3.17D0, ALFA=7.88D-2, CHROM=0.D0, SPRD=5.D-4
 &END
 &CPARM
 CRNT=4.0D0, STPC=4.0D0, NCR=100,  NMODF=-2, NMODE=1, KRAD=0
 IPRINT=.TRUE.
 &END
 &IPARM
 FREQ=1300.D0, RS=.4D0, QV=1.00D0
 &END
 &HPARM
 MMIN=-3, MMAX=1, TAUMIN=-.1, TAUMAX=.1
 &END
 &SPARM
 LMTRIX=.FALSE., LINTGL=.TRUE., LMAP=.FALSE., NDISP=0, NSTPS=1, SPRD0=0.,
 EX=0.1, EY=0.01, FEPS=1.0D-10, ZEPS=0.0, DXMAX=0.5, ESPC=0.001, CXMAX=0.5
 &END
```

Figure 1: A sample input data for MOSES.

```
*************************************************************************
*                                                                       *
*                              MOSES                                    *
*                                                                       *
*   MOde-coupling Single bunch instability in an Electron Storage ring  *
*                                                                       *
* D C I  of  O R S A Y - sample input data for MOSES                    *
* JOB-NAME:YOHMOSEM -DATE:19/04/88 -TIME:15.47.20   VERSION 2.0 (11/4/88) *
*                                                                       *
*************************************************************************


$MAPRM:
SYNCHROTRON TUNE              :  NUS     =  0.79200D-02
BEAM ENERGY                   :  ENGY    =  0.80000     (GeV)
SIGMA OF A GAUSSIAN BUNCH     :  SGMZ    =  -1.0000     (cm)        << BUNCH LENGTH VARIES WITH CURRENT >>
BETA-FUNCTION AT IMPEDANCE    :  BETAC   =   8.8600     (m)
REVOLUTION FREQUENCY          :  REVFRQ  =   3.1700     (MHz)
MOMENTUM COMPACTION FACTOR    :  ALFA    =  0.78800D-01
CHROMATICITY                  :  CHROM   =  0.0
BETATRON TUNE SPREAD AT SIGMA :  SPRD    =  0.50000D-03              << BETATRON TUNE HAS SPREAD >>

$IPARM:
RESONANT FREQUENCY            :  FREQ    =   1300.0     (MHz)
IMPEDANCE                     :  RS      =  0.40000     (MOhm/m)
Q - VALUE                     :  QV      =   1.0000

$CPARM:
STARTING CURRNT               :  CRNT    =   4.0000     (mA)
STEP IN CURRENT               :  STPC    =   4.0000     (mA)
NUMBER OF STEPS IN CURRENT    :  NCR     =   100
LOWEST AZIMUTHAL MODE         :  NMODF   =    -2
HIGHEST AZIMUTHAL MODE        :  NMODE   =     1
NUMBER OF LONG. RADIAL MODES  :  KRAD    =     0
PRINT OF EIGENVALUES          :  IPRINT  =     T

$HPARM:
MAXIMUM AZIMUTHAL MODE IN PLOT :  MMAX   =     1
MINIMUM AZIMUTHAL MODE IN PLOT :  MMIN   =    -3
MAXIMUM COORDINATE IN TAU PLOT :  TAUMAX =  0.10000
MINIMUM COORDINATE IN TAU PLOT :  TAUMIN = -0.10000
NUMBER OF SPLITTED PLOTS       :  NSPL   =     1

$SPARM:
DISPERSION MATRIX METHOD      :  LMTRIX  =     F
DISPERSION INTEGRAL METHOD    :  LINTGL  =     T
PLOT OF ABS(DETERMINANT)      :  LMAP    =     F
NUMBER OF TRANS. RADIAL MODES :  NDISP   =     0
NUMBER OF STEPS IN SPREAD     :  NSTPS   =     1
STARTING TUNE SPREAD          :  SPRD0   =  0.0
STEP IN REAL TUNE(=X) SEARCH  :  EX      =  0.10000
STEP IN IMAG TUNE(=Y) SEARCH  :  EY      =  0.10000D-01
CRITERION FOR CONVERGENCE OF F :  FEPS   =  0.10000D-09
CRITERION FOR CONVERGENCE OF Z :  ZEPS   =  0.0
MAXIMUM DIFFERENCE OF X       :  DXMAX   =  0.50000
MINIMUM DISTANCE BTWEEN TUNES :  ESPC    =  0.10000D-02
MAXIMUM CHANGE IN X           :  CXMAX   =  0.50000
```

Figure 2: Output for the test job shown in Fig.1

**********>>> BUNCH LENGTH VARIES WITH CURRENT AS FOLLOWS <<<**********

Current (mA)	/	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000
Bunch Length (cm)	/	4.964	6.254	7.159	7.880	8.488	9.020	9.495	9.928	10.325	10.694
Current (mA)	/	44.000	48.000	52.000	56.000	60.000	64.000	68.000	72.000	76.000	80.000
Bunch Length (cm)	/	11.039	11.364	11.672	11.963	12.242	12.508	12.763	13.009	13.245	13.474
Current (mA)	/	84.000	88.000	92.000	96.000	100.000	104.000	108.000	112.000	116.000	120.000
Bunch Length (cm)	/	13.695	13.909	14.116	14.318	14.514	14.705	14.891	15.073	15.250	15.424
Current (mA)	/	124.000	128.000	132.000	136.000	140.000	144.000	148.000	152.000	156.000	160.000
Bunch Length (cm)	/	15.593	15.759	15.922	16.081	16.237	16.390	16.540	16.688	16.833	16.976
Current (mA)	/	164.000	168.000	172.000	176.000	180.000	184.000	188.000	192.000	196.000	200.000
Bunch Length (cm)	/	17.116	17.254	17.390	17.524	17.656	17.786	17.914	18.040	18.164	18.287
Current (mA)	/	204.000	208.000	212.000	216.000	220.000	224.000	228.000	232.000	236.000	240.000
Bunch Length (cm)	/	18.408	18.527	18.645	18.762	18.877	18.991	19.103	19.214	19.324	19.433
Current (mA)	/	244.000	248.000	252.000	256.000	260.000	264.000	268.000	272.000	276.000	280.000
Bunch Length (cm)	/	19.540	19.646	19.751	19.855	19.958	20.060	20.161	20.261	20.359	20.457
Current (mA)	/	284.000	288.000	292.000	296.000	300.000	304.000	308.000	312.000	316.000	320.000
Bunch Length (cm)	/	20.554	20.650	20.745	20.840	20.933	21.026	21.118	21.209	21.299	21.388
Current (mA)	/	324.000	328.000	332.000	336.000	340.000	344.000	348.000	352.000	356.000	360.000
Bunch Length (cm)	/	21.477	21.565	21.652	21.739	21.825	21.910	21.995	22.079	22.162	22.245
Current (mA)	/	364.000	368.000	372.000	376.000	380.000	384.000	388.000	392.000	396.000	400.000
Bunch Length (cm)	/	22.327	22.408	22.489	22.570	22.649	22.729	22.807	22.885	22.963	23.040

****

**********>>> EIGEN VALUES (Nu-Nux)/Nus .VS. CURRENT <<<**********

BETATRON TUNE SPREAD = 0.0

Current (mA) /	4.0000		8.0000		12.0000		16.0000		20.0000	
Bunch length (cm) /	4.9638		6.2540		7.1590		7.8795		8.4880	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.00	:0.0	-2.00	:0.765E-11	-2.01	:0.0	-2.01	:0.0	-2.02	:0.0
	0.990	:0.0	0.976	:0.287E-10	-1.04	:0.0	0.951	:0.0	0.940	:0.0
	-1.01	:0.0	-1.02	:0.160E-10	-.188	:0.0	-.232	:0.0	-.272	:0.0
	-.790E-01	:0.0	-.138	:0.620E-11	0.962	:0.0	-1.05	:0.0	-1.06	:0.0

Current (mA) /	24.0000		28.0000		32.0000		36.0000		40.0000	
Bunch length (cm) /	9.0198		9.4954		9.9276		10.3251		10.6942	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.02	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.04	:0.0
	0.931	:0.0	0.922	:0.0	0.914	:0.0	0.906	:0.0	0.899	:0.0
	-.310	:0.0	-.345	:0.0	-.378	:0.0	-.410	:0.0	-.441	:0.0
	-1.07	:0.0	-1.08	:0.0	-1.09	:0.0	-1.10	:0.0	-1.10	:0.0

Current (mA) /	44.0000		48.0000		52.0000		56.0000		60.0000	
Bunch length (cm) /	11.0394		11.3643		11.6716		11.9635		12.2418	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.04	:0.0	-2.04	:0.0	-2.05	:0.0	-2.05	:0.0	-2.05	:0.0
	0.892	:0.0	0.885	:0.0	0.879	:0.0	0.872	:0.0	0.867	:0.0
	-.470	:0.0	-.499	:0.0	-.527	:0.0	-.554	:0.0	-.580	:0.0
	-1.11	:0.0	-1.12	:0.0	-1.13	:0.0	-1.14	:0.0	-1.14	:0.0

Current (mA) /	64.0000		68.0000		72.0000		76.0000		80.0000	
Bunch length (cm) /	12.5080		12.7633		13.0088		13.2454		13.4738	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.06	:0.0	-2.06	:0.0	-2.06	:0.0	-2.07	:0.0	-2.07	:0.0
	0.861	:0.0	0.855	:0.0	0.850	:0.0	0.845	:0.0	0.839	:0.0
	-.606	:0.0	-.631	:0.0	-.656	:0.0	-.680	:0.0	-.704	:0.0
	-1.15	:0.0	-1.16	:0.0	-1.16	:0.0	-1.17	:0.0	-1.18	:0.0

Current (mA) /	84.0000		88.0000		92.0000		96.0000		100.0000	
Bunch length (cm) /	13.6948		13.9088		14.1164		14.3181		14.5142	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.07	:0.0	-2.08	:0.0	-2.08	:0.0	-2.08	:0.0	0.816	:0.0
	0.834	:0.0	0.830	:0.0	0.825	:0.0	0.820	:0.0	-2.08	:0.0
	-.727	:0.0	-.750	:0.0	-.773	:0.0	-.795	:0.0	-.817	:0.0
	-1.18	:0.0	-1.19	:0.0	-1.20	:0.0	-1.20	:0.0	-1.21	:0.0

Current (mA) /	104.0000		108.0000		112.0000		116.0000		120.0000	
Bunch length (cm) /	14.7052		14.8914		15.0730		15.2504		15.4237	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	0.811	:0.0	0.807	:0.0	0.803	:0.0	0.798	:0.0	0.794	:0.0
	-2.09	:0.0	-2.09	:0.0	-2.09	:0.0	-2.10	:0.0	-2.10	:0.0
	-.839	:0.0	-.861	:0.0	-.882	:0.0	-.903	:0.0	-.924	:0.0
	-1.21	:0.0	-1.22	:0.0	-1.23	:0.0	-1.23	:0.0	-1.24	:0.0

Current (mA) /	124.0000		128.0000		132.0000		136.0000		140.0000	
Bunch length (cm) /	15.5932		15.7591		15.9216		16.0808		16.2369	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	0.790	:0.0	0.786	:0.0	0.782	:0.0	0.779	:0.0	0.775	:0.0

-2.10	:0.0	-2.11	:0.0	-2.11	:0.0	-2.11	:0.0	-2.12	:0.0
-.944	:0.0	-.965	:0.0	-.985	:0.0	-1.01	:0.0	-1.03	:0.0
-1.24	:0.0	-1.25	:0.0	-1.25	:0.0	-1.26	:0.0	-1.26	:0.0

Current (mA)	/	144.0000		148.0000		152.0000		156.0000		160.0000	
Bunch length (cm)	/	16.3901		16.5405		16.6882		16.8333		16.9760	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.771	:0.0	0.767	:0.0	0.764	:0.0	0.760	:0.0	0.757	:0.0
-2.12	:0.0	-2.12	:0.0	-2.13	:0.0	-2.13	:0.0	-2.13	:0.0
-1.05	:0.0	-1.07	:0.0	-1.09	:0.0	-1.11	:0.0	-1.13	:0.0
-1.27	:0.0	-1.27	:0.0	-1.27	:0.0	-1.28	:0.0	-1.28	:0.0

Current (mA)	/	164.0000		168.0000		172.0000		176.0000		180.0000	
Bunch length (cm)	/	17.1163		17.2543		17.3902		17.5240		17.6557	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.753	:0.0	0.750	:0.0	0.747	:0.0	0.743	:-.133E-07	0.740	:0.752E-07
-2.14	:0.0	-2.14	:0.0	-2.14	:0.0	-2.15	:0.278E-09	-2.15	:-.924E-07
-1.15	:0.0	-1.17	:0.0	-1.19	:0.0	-1.22	:0.284E-07	-1.25	:-.345E-07
-1.28	:0.0	-1.28	:0.0	-1.28	:0.0	-1.28	:-.210E-07	-1.27	:0.429E-07

Current (mA)	/	184.0000		188.0000		192.0000		196.0000		200.0000	
Bunch length (cm)	/	17.7855		17.9135		18.0397		18.1641		18.2868	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.737	:0.572E-07	0.734	:0.447E-07	0.731	:-.207E-07	0.728	:-.123E-06	0.725	:0.138E-06
-2.15	:0.173E-07	-2.16	:-.149E-07	-2.16	:0.177E-06	-2.16	:0.250E-06	-2.17	:0.261E-07
-1.27	:-.287E-01	-1.28	:-.411E-01	-1.29	:-.498E-01	-1.30	:-.567E-01	-1.31	:-.624E-01
-1.27	:0.287E-01	-1.28	:0.411E-01	-1.29	:0.498E-01	-1.30	:0.567E-01	-1.31	:0.624E-01

Current (mA)	/	204.0000		208.0000		212.0000		216.0000		220.0000	
Bunch length (cm)	/	18.4079		18.5274		18.6455		18.7620		18.8771	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.722	:0.261E-07	0.719	:-.212E-06	0.716	:-.373E-08	0.713	:0.112E-06	0.710	:0.253E-06
-2.17	:0.272E-06	-2.18	:0.570E-06	-2.18	:0.421E-06	-2.18	:0.365E-06	-2.19	:0.283E-06
-1.32	:-.671E-01	-1.33	:-.712E-01	-1.34	:-.746E-01	-1.35	:-.776E-01	-1.36	:-.802E-01
-1.32	:0.671E-01	-1.33	:0.712E-01	-1.34	:0.746E-01	-1.35	:0.776E-01	-1.36	:0.802E-01

Current (mA)	/	224.0000		228.0000		232.0000		236.0000		240.0000	
Bunch length (cm)	/	18.9908		19.1032		19.2143		19.3241		19.4326	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.707	:0.633E-07	0.704	:0.358E-06	0.702	:0.298E-06	0.699	:0.238E-06	0.696	:0.358E-06
-2.19	:0.235E-06	-2.20	:-.179E-06	-2.20	:-.179E-06	-2.21	:-.179E-06	-2.21	:-.119E-06
-1.37	:-.824E-01	-1.38	:-.842E-01	-1.39	:-.858E-01	-1.40	:-.871E-01	-1.41	:-.882E-01
-1.37	:0.824E-01	-1.38	:0.842E-01	-1.39	:0.858E-01	-1.40	:0.871E-01	-1.41	:0.882E-01

Current (mA)	/	244.0000		248.0000		252.0000		256.0000		260.0000	
Bunch length (cm)	/	19.5400		19.6462		19.7513		19.8552		19.9581	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.693	:0.358E-06	0.691	:0.179E-06	0.688	:0.298E-06	0.686	:0.417E-06	0.683	:0.238E-06
-2.21	:-.298E-06	-2.22	:-.119E-06	-2.22	:-.179E-06	-2.23	:-.179E-06	-2.23	:-.119E-06
-1.42	:-.890E-01	-1.43	:-.897E-01	-1.44	:-.901E-01	-1.45	:-.904E-01	-1.46	:-.905E-01
-1.42	:0.890E-01	-1.43	:0.897E-01	-1.44	:0.901E-01	-1.45	:0.904E-01	-1.46	:0.905E-01

Current (mA)	/	264.0000		268.0000		272.0000		276.0000		280.0000	
Bunch length (cm)	/	20.0599		20.1607		20.2605		20.3594		20.4572	
Synchrotron tune	/	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...		(REAL : IMAGINARY)									
0.680	:0.358E-06	0.678	:0.358E-06	0.675	:0.358E-06	0.673	:0.596E-07	0.671	:0.179E-06
-2.24	:-.119E-06	-2.24	:-.238E-06	-2.25	:-.119E-06	-2.25	:0.238E-06	-2.26	:-.119E-06

-1.47	:-.904E-01	-1.48	:-.902E-01	-1.49	:-.899E-01	-1.50	:-.894E-01	-1.50	:-.888E-01
-1.47	:0.904E-01	-1.48	:0.902E-01	-1.49	:0.899E-01	-1.50	:0.894E-01	-1.50	:0.888E-01

Current (mA) /	284.0000		288.0000		292.0000		296.0000		300.0000
Bunch length (cm) /	20.5542		20.6503		20.7454		20.8397		20.9332
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.668	:-.596E-07	0.666	:0.596E-07	0.663	:-.119E-06	0.661	:0.179E-06	0.659	:0.179E-06
-2.27	:0.238E-06	-2.27	:0.0	-2.28	:0.179E-06	-2.28	:0.596E-07	-2.29	:0.596E-07
-1.51	:-.881E-01	-1.52	:-.872E-01	-1.53	:-.863E-01	-1.54	:-.853E-01	-1.55	:-.841E-01
-1.51	:0.881E-01	-1.52	:0.872E-01	-1.53	:0.863E-01	-1.54	:0.853E-01	-1.55	:0.841E-01

Current (mA) /	304.0000		308.0000		312.0000		316.0000		320.0000
Bunch length (cm) /	21.0258		21.1176		21.2086		21.2989		21.3884
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.656	:0.0	0.654	:-.179E-06	0.652	:-.596E-07	0.650	:-.596E-07	0.647	:0.119E-06
-2.29	:0.119E-06	-2.30	:0.596E-07	-2.31	:0.596E-07	-2.31	:-.596E-07	-2.32	:-.179E-06
-1.55	:-.829E-01	-1.56	:-.816E-01	-1.57	:-.802E-01	-1.58	:-.787E-01	-1.58	:-.772E-01
-1.55	:0.829E-01	-1.56	:0.816E-01	-1.57	:0.802E-01	-1.58	:0.787E-01	-1.58	:0.772E-01

Current (mA) /	324.0000		328.0000		332.0000		336.0000		340.0000
Bunch length (cm) /	21.4771		21.5651		21.6525		21.7391		21.8250
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.645	:0.119E-06	0.643	:-.119E-06	0.641	:0.119E-06	0.639	:0.224E-06	0.637	:0.201E-06
-2.33	:0.596E-07	-2.33	:0.179E-06	-2.34	:-.596E-07	-2.35	:-.224E-06	-2.35	:-.142E-06
-1.59	:-.756E-01	-1.60	:-.739E-01	-1.61	:-.721E-01	-1.61	:-.703E-01	-1.62	:-.685E-01
-1.59	:0.756E-01	-1.60	:0.739E-01	-1.61	:0.721E-01	-1.61	:0.703E-01	-1.62	:0.685E-01

Current (mA) /	344.0000		348.0000		352.0000		356.0000		360.0000
Bunch length (cm) /	21.9102		21.9948		22.0788		22.1621		22.2448
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.635	:0.108E-06	0.632	:0.257E-06	0.630	:-.708E-07	0.628	:0.156E-06	0.626	:0.130E-06
-2.36	:0.112E-07	-2.37	:-.782E-07	-2.37	:0.261E-07	-2.38	:0.708E-07	-2.39	:0.484E-07
-1.63	:-.665E-01	-1.63	:-.646E-01	-1.64	:-.626E-01	-1.65	:-.605E-01	-1.66	:-.584E-01
-1.63	:0.665E-01	-1.63	:0.646E-01	-1.64	:0.626E-01	-1.65	:0.605E-01	-1.66	:0.584E-01

Current (mA) /	364.0000		368.0000		372.0000		376.0000		380.0000
Bunch length (cm) /	22.3269		22.4084		22.4893		22.5696		22.6493
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.624	:0.298E-07	0.622	:0.745E-07	0.620	:0.119E-06	0.618	:0.201E-06	0.616	:0.160E-06
-2.40	:0.559E-07	-2.41	:0.671E-07	-2.41	:-.373E-08	-2.42	:-.112E-06	-2.43	:-.224E-07
-1.66	:-.562E-01	-1.67	:-.540E-01	-1.67	:-.517E-01	-1.68	:-.494E-01	-1.69	:-.471E-01
-1.66	:0.562E-01	-1.67	:0.540E-01	-1.67	:0.517E-01	-1.68	:0.494E-01	-1.69	:0.471E-01

Current (mA) /	384.0000		388.0000		392.0000		396.0000		400.0000
Bunch length (cm) /	22.7285		22.8072		22.8853		22.9629		23.0399
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.615	:0.205E-06	0.613	:0.782E-07	0.611	:0.130E-06	0.609	:0.115E-06	0.607	:-.149E-07
-2.44	:0.373E-07	-2.45	:0.224E-07	-2.45	:-.373E-07	-2.46	:-.596E-07	-2.47	:0.969E-07
-1.69	:-.447E-01	-1.70	:-.422E-01	-1.71	:-.397E-01	-1.71	:-.372E-01	-1.72	:-.345E-01
-1.69	:0.447E-01	-1.70	:0.422E-01	-1.71	:0.397E-01	-1.71	:0.372E-01	-1.72	:0.345E-01

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**********>>> EIGEN VALUES (Nu-Nux)/Nus .VS. CURRENT <<<**********

BETATRON TUNE SPREAD = 0.50000D-03

Current (mA) /	4.0000		8.0000		12.0000		16.0000		20.0000	
Bunch length (cm) /	4.9638		6.2540		7.1590		7.8795		8.4880	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0
	0.812	:0.263E-10	0.740	:0.861E-10	-1.05	:-.191E-09	0.667	:0.127E-10	0.645	:-.164E-10
	-1.04	:-.282E-09	-1.05	:-.239E-10	-.515	:0.193E-09	-.571	:0.105E-09	-.617	:0.229E-09
	-.305	:0.0	-.439	:0.301E-09	0.697	:0.916E-10	-1.05	:-.165E-09	-1.04	:-.923E-10

Current (mA) /	24.0000		28.0000		32.0000		36.0000		40.0000	
Bunch length (cm) /	9.0198		9.4954		9.9276		10.3251		10.6942	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0
	0.628	:-.686E-10	0.613	:0.647E-10	0.600	:0.633E-10	0.589	:-.454E-10	0.578	:-.278E-10
	-.657	:0.136E-09	-.693	:-.863E-10	-.726	:0.100E-09	-.757	:0.615E-09	-.786	:-.982E-11
	-1.04	:-.115E-09	-1.03	:0.812E-10	-1.02	:0.256E-09	-1.02	:0.131E-09	-1.01	:-.194E-09

Current (mA) /	44.0000		48.0000		52.0000		56.0000		60.0000	
Bunch length (cm) /	11.0394		11.3643		11.6716		11.9635		12.2418	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0
	0.569	:-.224E-10	0.560	:-.652E-10	0.552	:-.257E-10	0.544	:-.242E-10	0.537	:0.328E-10
	-.814	:0.551E-09	-.841	:-.174E-09	-.868	:0.128E-09	-.893	:0.208E-09	-.918	:-.194E-09
	-1.01	:0.210E-09	-.841	:0.782E-10	-.868	:-.331E-09	-.893	:0.244E-09	-.918	:-.138E-09

Current (mA) /	64.0000		68.0000		72.0000		76.0000		80.0000	
Bunch length (cm) /	12.5080		12.7633		13.0088		13.2454		13.4738	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.03	:0.0	-2.03	:0.0	-2.03	:0.0	-2.02	:0.128E-09	-2.02	:0.703E-10
	0.530	:0.265E-10	0.523	:0.211E-11	0.517	:-.202E-10	0.511	:-.403E-10	0.505	:-.725E-11
	-.942	:0.341E-09	-.967	:0.330E-09	-1.00	:-.875E-09	-1.02	:0.126E-09	-1.03	:-.243E-09
	-.942	:0.118E-09	-.967	:0.326E-10	-1.00	:0.219E-09	-1.02	:-.242E-10	-1.03	:0.266E-09

Current (mA) /	84.0000		88.0000		92.0000		96.0000		100.0000	
Bunch length (cm) /	13.6948		13.9088		14.1164		14.3181		14.5142	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	-2.02	:-.814E-10	-2.02	:0.552E-10	-2.02	:-.750E-10	-2.02	:0.217E-10	0.479	:-.224E-10
	0.499	:-.183E-10	0.494	:0.250E-10	0.489	:0.444E-10	0.484	:0.216E-10	-2.02	:0.581E-10
	-1.05	:0.321E-09	-1.07	:0.458E-09	-1.10	:0.484E-09	-1.11	:0.159E-09	-1.13	:-.364E-09
	-1.05	:0.706E-09	-1.07	:0.431E-09	-1.10	:-.334E-09	-1.11	:0.121E-09	-1.13	:0.0

Current (mA) /	104.0000		108.0000		112.0000		116.0000		120.0000	
Bunch length (cm) /	14.7052		14.8914		15.0730		15.2504		15.4237	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	0.474	:-.203E-10	0.469	:0.216E-11	0.465	:-.121E-10	0.460	:-.181E-10	0.456	:0.293E-10
	-2.02	:-.125E-09	-2.02	:-.488E-10	-2.02	:0.521E-10	-2.02	:-.149E-09	-2.01	:-.132E-09
	-1.15	:-.570E-09	-1.17	:-.253E-09	-1.19	:0.204E-09	-1.21	:0.302E-03	-1.23	:0.219E-02
	-1.15	:0.261E-09	-1.17	:0.677E-09	-1.19	:-.365E-09	-1.21	:0.302E-03	-1.23	:-.219E-02

Current (mA) /	124.0000		128.0000		132.0000		136.0000		140.0000	
Bunch length (cm) /	15.5932		15.7591		15.9216		16.0808		16.2369	
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079	
(Nu-Nux)/Nus ...	(REAL	: IMAGINARY)								
	0.452	:-.263E-10	0.448	:-.136E-10	0.443	:0.407E-10	0.439	:0.302E-10	0.436	:-.113E-10

-2.01	:-.448E-10	-2.01	:-.111E-10	-2.01	:0.474E-10	-2.01	:0.154E-09	-2.01	:-.370E-09
-1.25	:-.399E-02	-1.26	:-.149E-01	-1.28	:0.158E-01	-1.30	:-.167E-01	-1.32	:0.176E-01
-1.25	:0.399E-02	-1.26	:0.149E-01	-1.28	:-.158E-01	-1.30	:0.167E-01	-1.32	:-.176E-01

Current (mA) /	144.0000		148.0000		152.0000		156.0000		160.0000
Bunch length (cm) /	16.3901		16.5405		16.6882		16.8333		16.9760
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.432	:0.313E-10	0.428	:-.556E-11	0.424	:-.897E-11	0.421	:0.125E-10	0.417	:-.255E-10
-2.01	:-.989E-10	-2.01	:0.188E-09	-2.01	:-.965E-10	-2.01	:-.147E-09	-1.94	:-.104E-01
-1.33	:-.185E-01	-1.35	:0.194E-01	-1.37	:-.204E-01	-1.38	:0.214E-01	-1.40	:-.224E-01
-1.33	:0.185E-01	-1.35	:-.194E-01	-1.37	:0.204E-01	-1.38	:-.214E-01	-1.40	:0.224E-01

Current (mA) /	164.0000		168.0000		172.0000		176.0000		180.0000
Bunch length (cm) /	17.1163		17.2543		17.3902		17.5240		17.6557
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.414	:0.633E-11	0.410	:-.227E-10	0.407	:0.128E-10	0.404	:-.290E-10	0.400	:0.184E-10
-1.67	:0.141	-2.33	:0.211E-09	-2.33	:-.228E-08	-2.33	:0.139E-08	-2.33	:-.822E-09
-1.42	:0.235E-01	-1.43	:-.246E-01	-1.45	:0.258E-01	-1.46	:-.270E-01	-1.48	:0.282E-01
-1.42	:-.235E-01	-1.43	:0.246E-01	-1.45	:-.258E-01	-1.46	:0.270E-01	-1.48	:-.282E-01

Current (mA) /	184.0000		188.0000		192.0000		196.0000		200.0000
Bunch length (cm) /	17.7855		17.9135		18.0397		18.1641		18.2868
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.397	:0.104E-10	0.394	:0.976E-11	0.391	:0.126E-10	0.388	:0.787E-11	0.385	:-.247E-10
-2.33	:0.250E-08	-2.33	:0.614E-09	-2.33	:-.108E-08	-2.33	:0.157E-08	-2.33	:0.302E-09
-1.50	:-.295E-01	-1.51	:0.309E-01	-1.53	:-.322E-01	-1.54	:0.336E-01	-1.55	:-.351E-01
-1.50	:0.295E-01	-1.51	:-.309E-01	-1.53	:0.322E-01	-1.54	:-.336E-01	-1.55	:0.351E-01

Current (mA) /	204.0000		208.0000		212.0000		216.0000		220.0000
Bunch length (cm) /	18.4079		18.5274		18.6455		18.7620		18.8771
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.382	:-.113E-10	0.379	:-.607E-11	0.376	:-.161E-10	0.373	:-.116E-10	0.371	:0.290E-10
-2.33	:-.724E-09	-2.33	:0.106E-08	-2.33	:0.572E-09	-2.33	:0.378E-09	-2.33	:0.504E-09
-1.57	:0.365E-01	-1.58	:-.380E-01	-1.60	:0.394E-01	-1.61	:-.409E-01	-1.62	:0.423E-01
-1.57	:-.365E-01	-1.58	:0.380E-01	-1.60	:-.394E-01	-1.61	:0.409E-01	-1.62	:-.423E-01

Current (mA) /	224.0000		228.0000		232.0000		236.0000		240.0000
Bunch length (cm) /	18.9908		19.1032		19.2143		19.3241		19.4326
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.368	:0.124E-10	0.365	:0.871E-11	0.362	:0.250E-10	0.360	:0.310E-10	0.357	:0.224E-10
-2.33	:0.972E-09	-2.33	:0.181E-08	-2.33	:0.303E-08	-2.33	:0.467E-08	-2.33	:-.120E-07
-1.63	:-.437E-01	-1.65	:0.450E-01	-1.66	:-.463E-01	-1.67	:0.475E-01	-1.68	:-.486E-01
-1.63	:0.437E-01	-1.65	:-.450E-01	-1.66	:0.463E-01	-1.67	:-.475E-01	-1.68	:0.486E-01

Current (mA) /	244.0000		248.0000		252.0000		256.0000		260.0000
Bunch length (cm) /	19.5400		19.6462		19.7513		19.8552		19.9581
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.355	:-.109E-10	0.352	:-.199E-10	0.350	:0.155E-10	0.347	:0.743E-12	0.345	:-.122E-10
-2.33	:0.838E-08	-2.33	:-.837E-08	-2.33	:-.120E-07	-2.33	:0.939E-08	-2.33	:-.747E-08
-1.69	:0.496E-01	-1.70	:-.505E-01	-1.72	:0.514E-01	-1.73	:-.521E-01	-1.74	:0.527E-01
-1.69	:-.496E-01	-1.70	:0.505E-01	-1.72	:-.514E-01	-1.73	:0.521E-01	-1.74	:-.527E-01

Current (mA) /	264.0000		268.0000		272.0000		276.0000		280.0000
Bunch length (cm) /	20.0599		20.1607		20.2605		20.3594		20.4572
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.342	:-.133E-10	0.340	:0.925E-11	0.337	:-.184E-11	0.335	:0.202E-10	0.333	:-.107E-10
-2.33	:-.135E-07	-2.33	:0.926E-08	-2.33	:-.776E-08	-2.33	:-.168E-07	-2.33	:0.786E-08

-1.75	:-.533E-01	-1.76	:0.537E-01	-1.77	:-.540E-01	-1.78	:0.542E-01	-1.78	:-.543E-01
-1.75	:0.533E-01	-1.76	:-.537E-01	-1.77	:0.540E-01	-1.78	:-.542E-01	-1.78	:0.543E-01

Current (mA) /	284.0000		288.0000		292.0000		296.0000		300.0000
Bunch length (cm) /	20.5542		20.6503		20.7454		20.8397		20.9332
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.330	:0.154E-11	0.328	:-.135E-10	0.326	:-.165E-10	0.324	:-.117E-10	0.321	:0.341E-12
-2.33	:-.943E-08	-2.33	:-.296E-08	-2.33	:-.510E-08	-2.59	:-.987E-01	-2.59	:-.421E-01
-1.79	:0.544E-01	-1.80	:-.543E-01	-1.81	:0.541E-01	-1.82	:-.539E-01	-1.83	:0.536E-01
-1.79	:-.544E-01	-1.80	:0.543E-01	-1.81	:-.541E-01	-1.82	:0.539E-01	-1.83	:-.536E-01

Current (mA) /	304.0000		308.0000		312.0000		316.0000		320.0000
Bunch length (cm) /	21.0258		21.1176		21.2086		21.2989		21.3884
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.319	:-.804E-11	0.317	:0.108E-10	0.315	:0.309E-10	0.313	:-.106E-10	0.311	:-.152E-10
-2.56	:0.360E-09	-2.57	:-.131E-09	-2.58	:-.182E-09	-2.59	:0.671E-10	-2.60	:0.485E-11
-1.84	:-.532E-01	-1.84	:0.527E-01	-1.85	:-.521E-01	-1.86	:0.515E-01	-1.87	:-.508E-01
-1.84	:0.532E-01	-1.84	:-.527E-01	-1.85	:0.521E-01	-1.86	:-.515E-01	-1.87	:0.508E-01

Current (mA) /	324.0000		328.0000		332.0000		336.0000		340.0000
Bunch length (cm) /	21.4771		21.5651		21.6525		21.7391		21.8250
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.309	:-.162E-10	0.307	:0.729E-11	0.305	:-.110E-10	0.303	:-.362E-11	0.301	:0.968E-11
-2.60	:0.133E-09	-2.61	:0.135E-09	-2.62	:0.843E-10	-2.63	:-.187E-09	-2.64	:-.212E-09
-1.88	:0.500E-01	-1.88	:-.492E-01	-1.89	:0.483E-01	-1.90	:-.473E-01	-1.90	:0.464E-01
-1.88	:-.500E-01	-1.88	:0.492E-01	-1.89	:-.483E-01	-1.90	:0.473E-01	-1.90	:-.464E-01

Current (mA) /	344.0000		348.0000		352.0000		356.0000		360.0000
Bunch length (cm) /	21.9102		21.9948		22.0788		22.1621		22.2448
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.299	:-.400E-10	0.297	:0.422E-11	0.295	:0.110E-10	0.293	:-.776E-12	0.291	:0.256E-10
-2.65	:0.184E-09	-2.66	:0.962E-10	-2.66	:0.102E-09	-2.67	:0.835E-10	-2.68	:0.659E-10
-1.91	:-.453E-01	-1.92	:0.443E-01	-1.92	:-.431E-01	-1.93	:0.420E-01	-1.94	:-.408E-01
-1.91	:0.453E-01	-1.92	:-.443E-01	-1.92	:0.431E-01	-1.93	:-.420E-01	-1.94	:0.408E-01

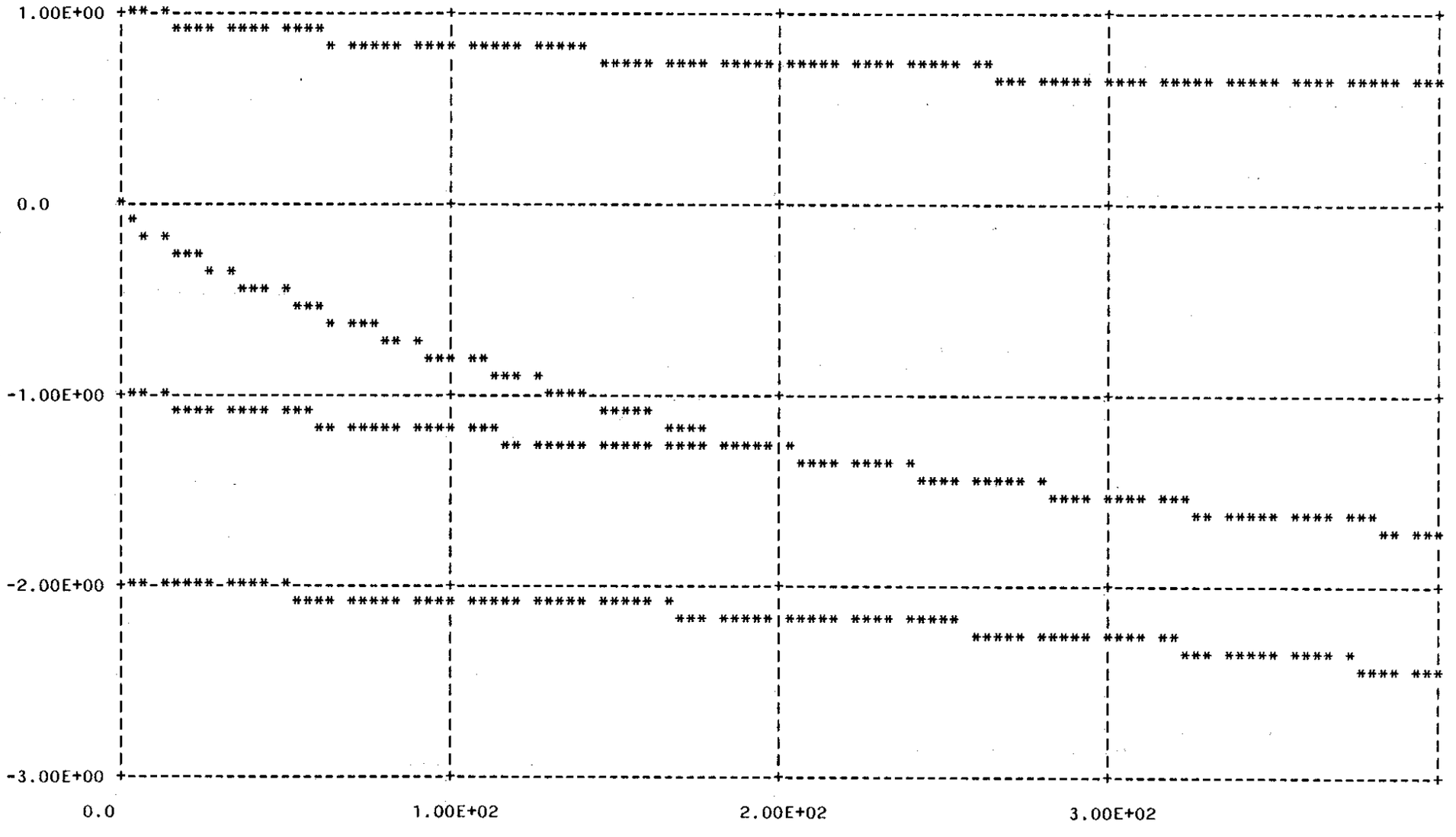
Current (mA) /	364.0000		368.0000		372.0000		376.0000		380.0000
Bunch length (cm) /	22.3269		22.4084		22.4893		22.5696		22.6493
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.289	:-.119E-10	0.287	:0.730E-11	0.286	:0.508E-11	0.284	:0.111E-10	0.282	:-.123E-11
-2.69	:-.203E-09	-2.70	:0.601E-10	-2.71	:-.147E-09	-2.72	:0.924E-10	-2.72	:-.417E-10
-1.94	:0.396E-01	-1.95	:-.383E-01	-1.95	:0.370E-01	-1.96	:-.357E-01	-1.96	:0.343E-01
-1.94	:-.396E-01	-1.95	:0.383E-01	-1.95	:-.370E-01	-1.96	:0.357E-01	-1.96	:-.343E-01

Current (mA) /	384.0000		388.0000		392.0000		396.0000		400.0000
Bunch length (cm) /	22.7285		22.8072		22.8853		22.9629		23.0399
Synchrotron tune /	0.0079		0.0079		0.0079		0.0079		0.0079
(Nu-Nux)/Nus ...	(REAL : IMAGINARY)								
0.280	:0.161E-11	0.278	:-.986E-10	0.277	:0.264E-10	0.275	:0.291E-11	0.273	:-.878E-10
-2.73	:0.857E-10	-2.74	:0.142E-09	-2.75	:0.988E-11	-2.76	:-.106E-09	-2.77	:0.477E-10
-1.97	:-.329E-01	-1.97	:0.315E-01	-1.98	:-.300E-01	-1.98	:0.285E-01	-1.99	:-.269E-01
-1.97	:0.329E-01	-1.97	:-.315E-01	-1.98	:0.300E-01	-1.98	:-.285E-01	-1.99	:0.269E-01

****

PROBLEM = D C I of O R S A Y - sample input data for MOSES 19/04/88 15.47.20 SPRD = 0.0

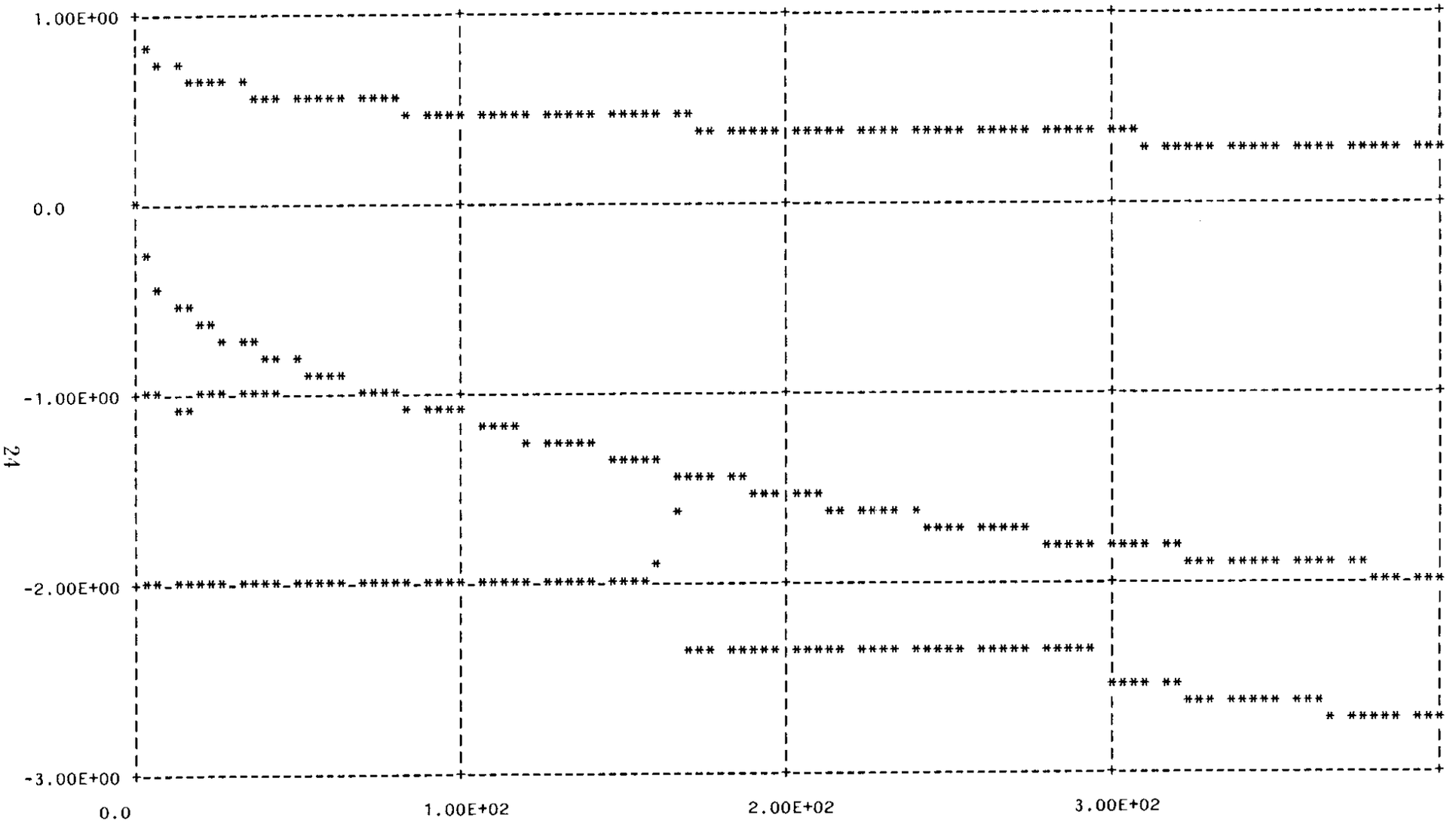
(*) = Real part of (Nu-Nux)/Nus as a function of bunch current in mA



PROBLEM = D C I of O R S A Y - sample input data for MOSES 19/04/88 15.47.20 SPRD = 0.5000D-03

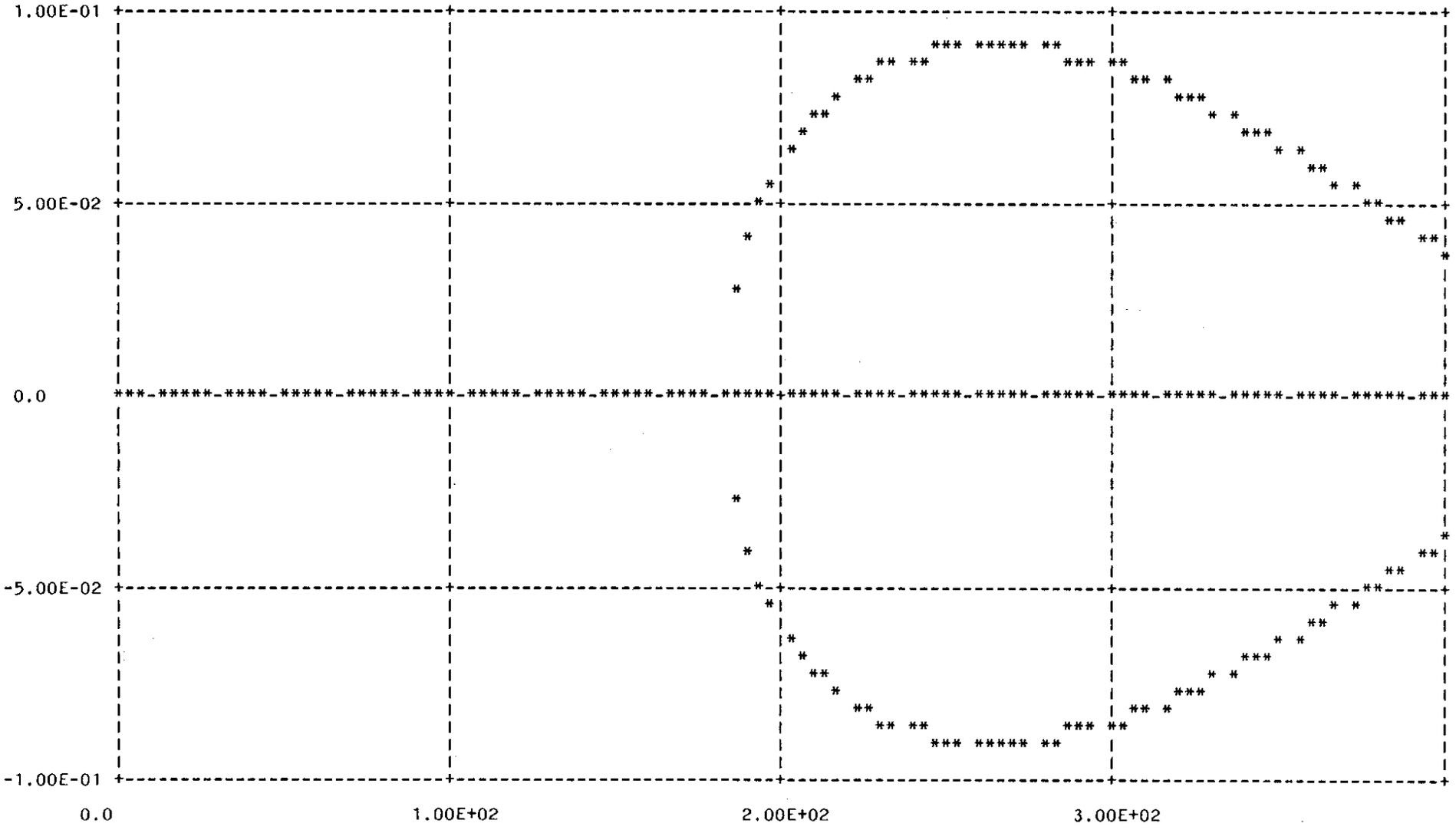
(*) = Real part of (Nu-Nux)/Nus as a function of bunch current in mA

<<< THE DISPERSION INTEGRAL METHOD >>>



PROBLEM = D C I of O R S A Y - sample input data for MOSES 19/04/88 15.47.20 SPRD = 0.0

(*) = Imaginary part of (Nu-Nux)/Nus as a function of bunch current in mA



PROBLEM = D C I of O R S A Y - sample input data for MOSES 19/04/88 15.47.20 SPRD = 0.5000D-03

(*) = Imaginary part of (Nu-Nux)/Nus as a function of bunch current in mA

<<< THE DISPERSION INTEGRAL METHOD >>>